CHAPTER V

STRUCTURAL DESIGN AND ANALYSIS

5.1 INTRODUCTION

The design and analysis of structural components subjected to dynamic loads differs from conventional static design procedures in that the time varying characteristics of the loading and the inertial characteristics of the structure must be considered. Following paragraphs of this chapter describe techniques which are sufficiently accurate for preliminary designs in all cases, and in most cases, adequate for final designs. These methods deal primarily with the dynamic loadings imposed by internal explosions.

The type of operation and the explosive size and characteristic to be contained within the suppressive shield dictate configuration and dimensions for the structure. With the explosive data and structural dimensions established, the internal airblast environment and fragment hazard can be defined using the procedures presented in Chapter 3. Safety criteria determine the allowable venting ratio. The internal airblast pressures determine the strength of structural elements and the fragment weights and velocities often determine the minimum thickness of these elements.

Structural design to resist dynamic loads is an iterative procedure. After an explosive environment is defined for a suppressive shield element, a trial structural section is selected to perform the first design calculation. If required by the first design calculation, the trial section is modified and used as the trial section for the second design calculation. The process is repeated until the resistance of the selected section is equal to or slightly greater than the required resistance. Very seldom are more than three iterations needed.

On the other hand structural analysis of existing structures does not always require iterations. A closed form solution is used to obtain structural deformation or ductility ratios for a specified loading. If the objective of the analysis is to determine the maximum explosive charge an existing structure can withstand, an iterative process is still required.

Before proceeding with either design or analysis discussed above, a decision must have been made as to what damage to the structure is acceptable. Damages are measured by ductility ratios. Chapter 4 defines, discusses and recommends acceptable ductility ratios. If the recommended ductility ratios are used, the safety criteria for containment of airblast, fragments, and fireball will be met.

5.2 STRUCTURAL RESISTANCE

As noted in Chapter 4, most suppressive shield structural elements are designed under the assumption that some inelastic response is acceptable and desirable. For these elements, the displacement-resistance function is nonlinear and is assumed to be represented by one of the idealized functions shown in Fig. 5-1. Resistance is proportional to displacement up to the point of yielding. Beyond the point of yielding, the resistance of the element may increase, remain constant or even decrease. An increase in resistance may result from strain hardening of the material or the development of membrane action in the element. A decaying resistance might be the result of local buckling or axial compressive loads on the element. This type of resistance-displacement function is undesirable and can normally be avoided by proper design of the structural system. Neglect of strain hardening or membrane action results in a more conservative design; however, it is often difficult to ascertain exactly how much benefit might accrue from these effects. The elastic-plastic resistance functions used in this handbook neglect any enhancement or degradation of structure resistance from the above effects.

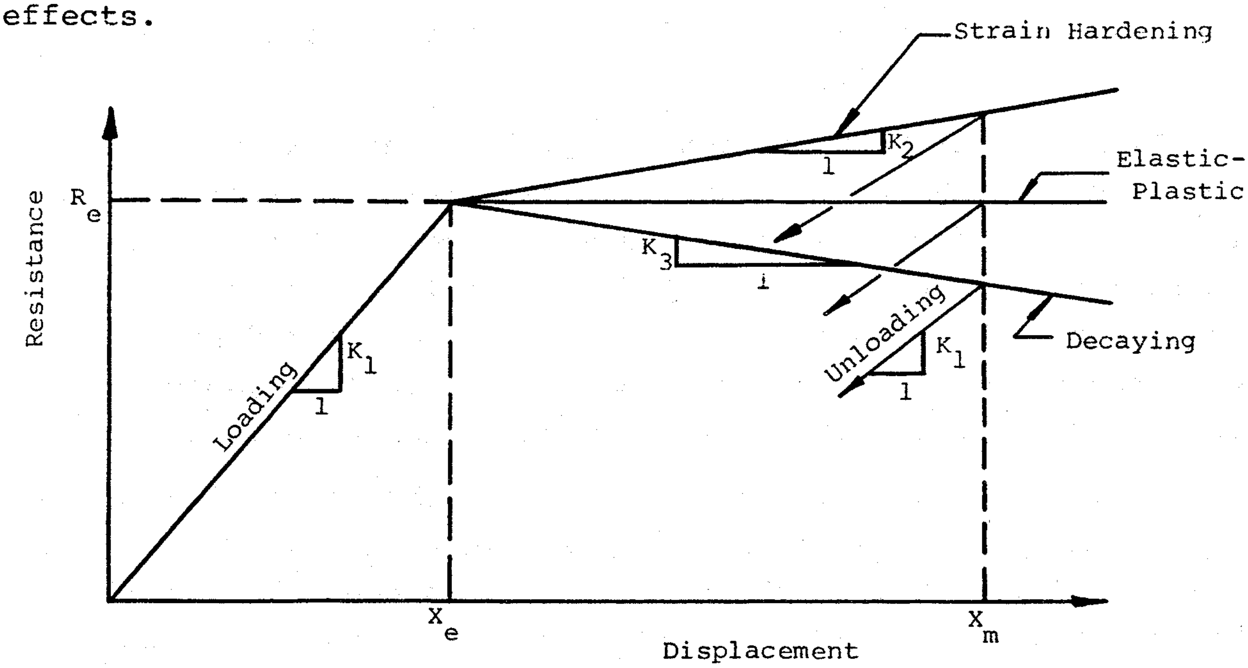


Figure 5-1. Idealized Resistance Functions

For elastic-plastic systems, the response is elastic up to the elastic limit X_e. The resistance then remains constant over the displacement range, $X_e \leq X \leq X_m$, where X_m is the maximum displacement. As the displacement starts to decrease, the response or rebound is again assumed to be elastic.

The resistance functions shown in Fig. 5-1 are representative of the idealized elastic-plastic behavior of statically determinate structures such as a simple beam. That is, as the load (assumed uniform) is increased on the beam, the displacement at midspan increases to some value X_e at which point the moment capacity of the beam has been reached and a plastic hinge forms at midspan. Assuming perfectly plastic behavior, the displacement can now increase indefinitely with no further increase in load.

Statically indeterminate structures possess additional load carrying capacity beyond formation of the first plastic hinge(s). A uniformly loaded beam with both ends fixed would have a resistance function similar to that shown in Fig. 5-2. As the load increases, the moments at the fixed supports increase until the plastic moment capacity of the beam is reached and plastic hinges form. This portion of the resistance function is that shown in Fig. 5-2 as zero to 1.

Although the beam section has yielded and plastic hinges have formed at the fixed supports, the member is still capable of supporting increased load as a simple beam. This portion of the resistance function is that from 1 to 2 in Fig. 5-2. Point 2 represents the formation of a plastic hinge at midspan which converts the beam into a mechanism theoretically capable of increasing deflection without limit with no increase in load.

It is frequently found convenient in the accomplishment of simplified dynamic analyses to replace the bilinear curve 0-1-2 in Fig. 5-2 with the single line 0-3. This equivalent resistance function can be constructed by equating the areas under the actual and equivalent curves. The equivalent deflection X_E can be found by

$$X_E = X_e + X_p(1 - R_e/R_m) \quad (5\text{-}1)$$

and the equivalent stiffness K_E of the system by

$$K_E = R_m/X_E \quad (5\text{-}2)$$

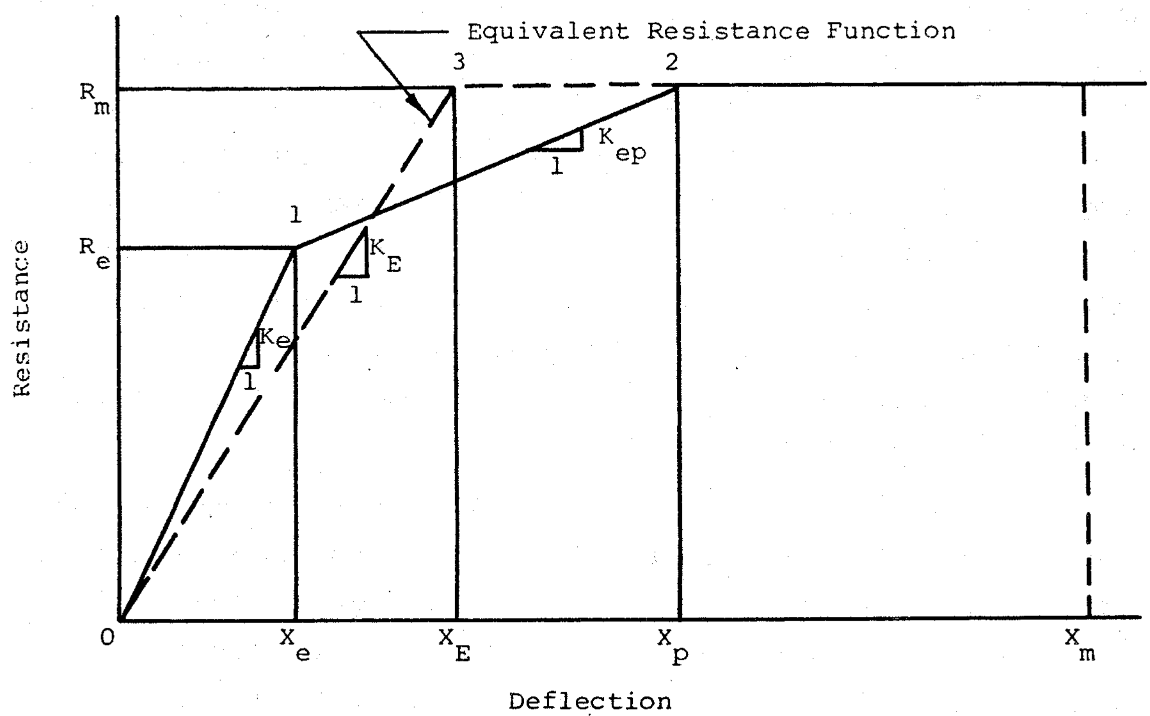


Figure 5-2. Idealized Resistance Function for Uniform Loaded Fixed End Beam

The curves shown in Fig. 5-2 are elastic-perfectly plastic, i.e., they contain elastic portions with a linear relationship between load and deflection and perfectly plastic portions where indefinite deflection is possible at constant load. When very large deflections (strains) are considered, the elastic portion of the resistance can be neglected and the behavior considered to be rigid-plastic (i.e., the resistance function could be taken as a horizontal line with ordinate R_m) with little error.

5.3 PROPERTIES OF STRUCTURAL ELEMENTS

5.3.1 General

The design or analysis of structures and structural elements to resist dynamic loads requires a determination of the static load carrying capacity of the element. A dynamic analysis is performed to obtain a required static resistance or to translate a given static resistance into one under dynamically applied loads. This section presents conventional expressions for the flexural, shear and axial load capacity of steel and reinforced concrete elements. These expressions are utilized in paragraph 5.4 to obtain the resistances of beams and slabs with various end conditions and span ratios. Also presented are expressions for the load capacity of cylindrical and spherical pressure vessels. As noted in Chapter 4, dynamic tensile or compressive strengths should be used to obtain the strength of elements subjected to dynamic loadings.

The design or analysis of structural members for suppressive shields will almost always be based upon inelastic behavior of the member. For steel, the design procedure is referred to as plastic design; for concrete, it is ultimate strength design. These methods assume both ultimate strength behavior (plastic moments) and the redistribution of load due to formation of plastic hinges.

5.3.2 Structural Steel Elements

In designing or analyzing the ability of steel members to resist blast effects, many of the concepts and equations developed for the plastic analysis of steel structures under static loads are used. A number of references (such as Refs. 5-1 and 5-2) contain discussions of plastic analysis and design of steel structures for static loads and can be consulted for more detailed guidance.

a. Flexure

If a steel member is subjected to pure bending, its ultimate moment capacity is given by

$$M_p = f_{dy} Z \tag{5-3}$$

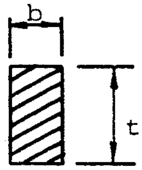
where Z is the plastic section modulus of the member and f_{dy} is the dynamic yield strength of the steel. Reference 5-2 includes plastic section modulus tables for common structural steel sections, and Fig. 5-3 gives general expressions for the plastic section modulus of several structural shapes for bending about a horizontal centroidal axis.

Equation 5-3 assumes that the member is properly supported and proportioned so as to allow development of a plastic hinge at critical sections. If the member is not properly supported or proportioned, buckling may occur before the fully plastic moment can be developed. To ensure the ability of a steel member to sustain fully plastic hinge formation, it is necessary that the member be properly braced to prevent lateral-torsional buckling and that the elements of the member meet minimum thickness requirements for initial loading and rebound (see Ref. 5-3). Table 5-1 gives maximum width-thickness ratios for flanges of rolled, wide-flange shapes and similar built-up single-web shapes that are subjected to compression involving hinge rotation under ultimate loading.

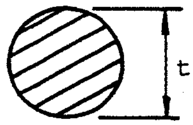
b. Shear

Shear is of interest in steel members primarily because of its possible influence on the plastic moment capacity of the member. It has been found experimentally that the member will achieve its full plastic moment capacity if the average shear stress over the full web area is less than the yield stress in shear (Ref. 5-1).

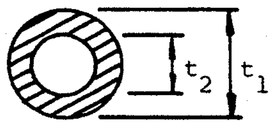
From Ref. 5-2, the shear capacity of WF or I-shaped steel sections with unstiffened webs is given by



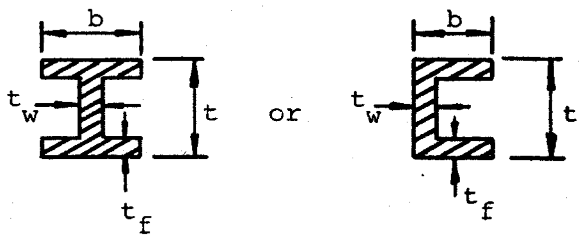
$Z = bt^2/4$



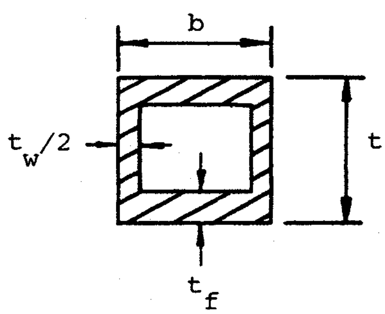
$Z = t^3/6$



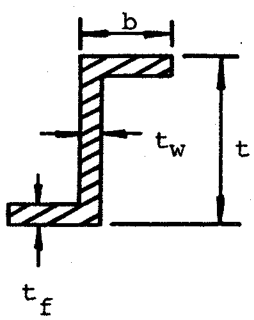
$Z = \frac{1}{6}\left(t_1^3 - t_2^3\right)$



$Z = tt_w\left(\frac{t}{4} - t_f\right) + bt_f\left(t-t_f\right) + t_w t_f^2$

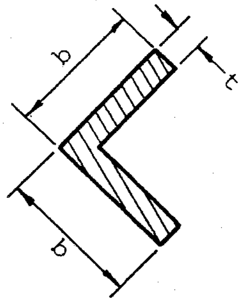


Z = same as wide flange or channel

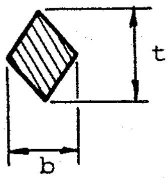


Z = same as wide flange or channel

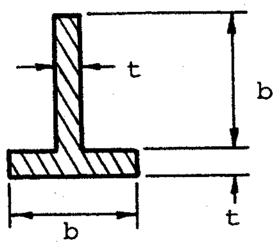
Figure 5-3a. Plastic Section Moduli for Structural Shapes



$$Z \approx \frac{b^2 t}{\sqrt{2}}$$

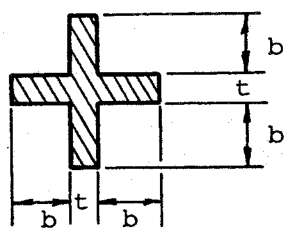


$$Z = \frac{bt^2}{12}$$



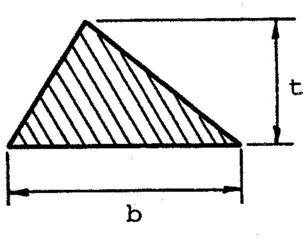
$$Z = \frac{b^2 t}{2}$$

$t \ll b$



$$Z \approx \frac{bt}{2}\,(2b + 3t)$$

$t \ll b$



$$Z = 0.0976bt^2$$

Figure 5-3b. Plastic Section Moduli for Structural Shapes (concluded)

Table 5-1

MAXIMUM THICKNESS RATIOS FOR STEEL MEMBERS (Ref. 5-2)

f_y, ksi	$b/2t_f$	d_w/t_w
36	8.5	43
42	8.0	40
45	7.4	38
50	7.0	36
55	6.6	35
60	6.3	33
65	6.0	32

b = width of flange; t_f = thickness of flange (if thickness of flange varies, use average thickness); d_w = depth of web; t_w = thickness of web.

$$V_u = 0.55 f_{dy} t_w t \tag{5-4}$$

where

V_u = ultimate shear capacity

t_w = web thickness

t = total depth of member

When the web of a built-up section is designed to carry a significant part of the total moment requirement of the section, the shear influence cannot be neglected and the member should be investigated for possible moment capacity loss through shear yield. Reference 5-3 recommends that the moment capacity of such a section be defined by

$$M_p = b f_{dy} t_f d_w \left\{ 1 + 0.25 \left(\frac{t_w d_w}{t_f b} \right) \left[1 - \left(\frac{V}{V_y} \right)^2 \right] \right\} \tag{5-5}$$

where

b = flange width

t_f = flange thickness

d_w = depth of web = $t - 2t_f$

V = total shear acting on section

V_y = $0.55 f_{dy} t_w d_w$ = shear capacity of web

c. Axial Loads

Due to the nature of suppressive shield structural configurations (i.e., loaded from the interior such as pressure vessels), compressive axial loads will rarely, if ever, be a consideration. Even with those configurations that utilize columns, such columns properly proportioned for the rated blast loads will almost certainly be adequate for the normal static service loads and rebound loads. Such adequacy can be readily verified by the procedures presented in Refs. 5-1 and 5-2.

Tensile axial loads can also reduce the moment capacity of steel members. However, columns and roof beams proportioned for moment due to blast loads by the methods of this handbook are not expected to experience any significant reduction in load-carrying capability due to combined tensile and flexural forces. Reference 5-1 or 5-3 is recommended should the occasion arise to investigate the effects of combined axial and flexural forces on steel members.

5.3.3 Reinforced Concrete Elements

The only reinforced concrete structural elements of potential interest for suppressive shields are beams and slabs (plates). These elements may be utilized for suppressive shield foundations or roof slabs. The use of reinforced concrete in cylindrical and sperical structural configurations is not recommended for suppressive shielding applications.

Ultimate strength design methods are used for reinforced concrete elements, and a properly designed and proportioned reinforced concrete member is theoretically as ductile in flexure as a structural steel member. If reinforced concrete members are used, they should be Type I construction as defined in Chapter 4.

a. Flexure

The flexural mode of response is heavily dependent upon the percentage of tensile steel employed. If insufficient steel is used, the steel may be incapable of resisting the tensile force carried by the concrete before cracking. If, on the other hand, an excessively large percentage of steel is used, the concrete crushes on the compression side before the tensile steel yields. To avoid either of these undesirable characteristics and to ensure ductile response, reinforced concrete flexural members with tensile reinforcing only should be proportioned so that (Ref. 5-4)

$$p \leq \frac{55,463\ B_1}{87,000 + f_y}\left[\frac{f'_c}{f_y}\right] \tag{5-6}$$

where

p = tensile reinforcing steel ratio = A_s/bd

A_s = cross sectional area of tensile steel

b = width of concrete member

d = effective depth of concrete member (Distance from the compression outer fiber to the centroid of the tensile reinforcing steel)

f'_c = static unconfined compressive strength of concrete

f_y = static yield strength of steel

B_1 = .85 for $f'_c \leq$ 4,000 psi and is reduced at a rate of 0.05 for each 1,000 psi increase in f'_c over 4,000 psi.

The ultimate moment capacity of a rectangular member with tensile reinforcing only and subjected to bending only is given by

$$M_p = pf_{dy}bd^2(1 - 0.59pf_{dy}/f'_{dc}) \tag{5-7}$$

where f'_{dc} is the dynamic compressive strength of the concrete and all other terms are as previously defined.

The addition of compression steel has little effect on the ultimate moment capacity of underreinforced members (those meeting the criteria of Eq. 5-6). It is recommended that the member be proportioned according to Eq. 5-7 and that for small shields, 25% of the rebars be conservatively provided on the opposite face for rebound resistance but that no increase in moment capacity be claimed due to the presence of reinforcing steel in the compression area. If there is found to be some overriding reason to take account of the effects of compression reinforcement, Ref. 5-4 or 5-5 should be consulted. If the required rebound resistance is determined by dynamic analysis, the reinforcing steel in the opposite face should provide this resistance. See Section 5.5.5 (to be added) for rebound calculations.

It is often necessary to calculate the moment of inertia of a reinforced concrete element. Reference 5-6 recomments that the moment of inertia be taken equal to the average of that for the cracked and uncracked transformed cross sections. For rectangular cross sections, Ref. 5-6 recommends the approximation

$$I_a = \frac{bd^3}{2} (5.5p + 0.083) \qquad (5\text{-}8)$$

b. Shear

Shear failures are generally brittle in nature with little advance warning of distress in the member. In order to assure ductile behavior of reinforced concrete members, it is necessary that members be designed against shear failure by following recommended ductility ratios in Table 4-3. The static compressive allowable stress should be used in expressions for the shear strength of reinforced concrete members.

There are two modes of shear failure, direct shear and diagonal tension. The direct shear mode of failure is characterized by the rapid propagation of a nearly vertical crack through the depth of the member in the region of the support. Horizontal reinforcement inhibits the formation and progagation of such cracks. Direct shear failures can occur in members properly proportioned for suppressive shielding applications. Direct shear should always be investigated. The direct shear stress that can be taken by the concrete is given by (Ref. 5-5)

$$v_d = \frac{V_d}{bd} = 0.18f'_c \text{ psi} \qquad (5\text{-}9)$$

where V_d is the total shear at the support in pounds and the other terms are as previously defined. Equation 5-9 may be used for either conventional (span/depth > 5) or deep (span/depth < 5) members.

The diagonal tension failure mode is characterized by diagonal cracks which propagate through the member from a point near the tensile steel toward the compression face. When the crack has penetrated to a point where the remaining compression zone of the concrete is insufficient to sustain the bending stresses, the concrete crushes and the member fails.

The critical section for diagonal tension in conventional members is taken at a distance d (the effective depth of the member) from the support. The allowable shear stress on the concrete is given by (Ref. 5-7)

$$v_c = (1.9\sqrt{f'_c} + 2500 p d V_c / M_c) \qquad (5\text{-}10)$$

where

V_c = total shear on critical section (typically at distance d from the support)

M_c = moment at the critical section

The value of the term dV_c/M_c in Eq. 5-10 shall not be taken greater than 1.0. Reference 5-7 states that the shearing stress obtained from Eq. 5-10 should not exceed $3.5\sqrt{f'_c}$. Reference 5-5 recommends a more conservative value of $2.28\sqrt{f'_c}$ A review of test data reported in Ref 5-8 indicates that the value of $2.28\sqrt{f'_c}$ is perhaps overly conservative.

The added shear capacity contributed by shear reinforcing is given by (Ref. 5-9)

$$V_s = d \frac{A_v f_{dy}}{s} \qquad (5\text{-}11)$$

where

s = spacing of vertical web reinforcing

A_v = total cross section area of web reinforcing over distance s

The vertical web reinforcing ratio is defined as the ratio of the area of the vertical web reinforcing to the gross horizontal area, bs. Equation 5-11 assumes the web reinforcing is placed perpendicular to the longitudinal axis of the member. Reference 5-7 states that V_s/bd should not exceed

$8\sqrt{f'_c}$ psi. The total shear capacity is then given by

$$V_u = V_c + V_s \tag{5-12}$$

where V_c equals the allowable concrete shear stress from Eq. 5-10, v_c times bd. The shear stress calculated from Eq. 5-12, i.e., V_u/bd, should not exceed $11.5\sqrt{f'_c}$ psi. Reference 5-5 recommends a more conservative value of $10\sqrt{f'_c}$.

The critical section for diagonal tension (shear) in deep members is assumed to occur at a distance 0.15L from the support for uniformly loaded members, one-half the distance between a concentrated load and the support for concentrated loads, but not over a distance d from the support for either case. The allowable shear stress on the concrete for deep members is given by (Ref. 5-8)

$$v_c = (3.5 - 2.5M_c\ /V_c\ d)(1.9\sqrt{f'_c} + 2500pdV_c\ /M_c\ ) \tag{5-13}$$

with the provisions that

$$1.0 \leq (3.5 - 2.5M_c\ /V_c\ d) \leq 2.5$$

and

$$v_c \leq 6\sqrt{f'_c}$$

with all terms as previously defined.

When web reinforcing is needed to supply additional shear capacity for deep members, it is recommended that such reinforcement be provided by an orthogonal vertical and horizontal system of bars. The shear capacity contributed by such a system is given by (Ref. 5-7)

$$V_s = f_{dy}d\left[\frac{A_v}{12s}\left(1 + \frac{L}{d}\right) + \frac{A_{vH}}{12s_H}\left(11 - \frac{L}{d}\right)\right] \tag{5-14}$$

where

A_v = total cross section area of vertical web reinforcing over distance s

s = horizontal spacing of vertical web reinforcing

L = span of member

A_{vH} = total cross section area of horizontal web reinforcing over distance s_H

s_H = vertical spacing of horizontal web reinforcing

The total shear stress, i.e., V_u/bd with V_u from Eq. 5-12, to be allowed on a deep member shall be limited to $10\sqrt{f'_c}$ psi.

The web reinforcing systems described above are the conventional methods of providing shear reinforcement for reinforced concrete members. Where shear reinforcement is required for conventional members, the amount of such reinforcement provided shall be

$$A_v \geq \frac{50bs}{f_{dy}}$$

and s shall not exceed d/2 or 24 inches. Where required in deep members, the area of shear reinforcement A_v perpendicular to the main reinforcement shall be not less than 0.0015bs and s shall not exceed d/5 or 18 inches. The area of shear reinforcement A_{vH} parallel to the main reinforcement shall not be less than, $0.0025bs_H$ and s_H shall not exceed d/3 or 18 inches.

The situation may arise where it is desirable to utilize a reinforced concrete element where the loading conditions are such that the allowable total shear values stated above ($V_u/bd \leq 11.5\sqrt{f'_c}$ or $\leq 10\sqrt{f'_c}$) are exceeded. In such a case, increase the depth of the member.

c. Bond and Anchorage

All modes of failure of reinforced concrete elements are closely coupled to and are, in fact, inseparable from a bond mode of failure. If a bond failure is not prevented, the bars will not serve their function in the other modes of behavior considered.

The tension or compression forces in the reinforcement at each section must be developed on each side of that section by an adequate embedment length or end anchorage or a combination of the two. If no mechanical end anchorage is provided, the tension or compression forces in the reinforcing must be resisted by shear-type bond stresses distributed over the contact area between the bars and the concrete. Bars without deformations shall not be used. The projecting ribs of deformed bars bear against the surrounding concrete and provide greatly increased bond strength over that of plain bars.

Reference 5-4 states that the ultimate resisting bond force, in force per unit length of bar, is largely independent of bar size or perimeter. Since the force in the bar causing bond failure increases with its area, bond is a more serious problem with the larger bars. The critical sections for development of reinforcement in flexural members are generally at points of maximum moment gradient. The required development length of deformed bars in tension is given by (Ref. 5-7)

$$L_D = 0.04Af_{dy}/\sqrt{f'_c}$$

but not less than (5-15)

$$0.0004Df_{dy}$$

where L_D is in inches, A is the cross section area of an individual bar in square inches and D is the diameter of the bar in inches. If the reinforcement is placed horizontally in the top of a member with more than 12 inches of concrete below it, the values obtained from Eq. 5-15 are multiplied by 1.4. For reinforcement whose f_{dy} is greater than 60,000 psi, the values obtained from Eq. 5-15 are multiplied by the factor $\left[2-(60{,}000/f_{dy})\right]$. Equation 5-15 is limited to #11 reinforcing bars and smaller. Bars larger in diameter than #11 are not recommended for suppressive shielding applications.

The development length for bars in compression is given by

$$L_D = 0.02 f_{dy} D/\sqrt{f'_c} \tag{5-16}$$

but not less than

$$0.0003 f_{dy} D \text{ or } 8 \text{ inches}$$

Additional guidance on development of bond strength is presented in Ref. 5-7.

d. Axial Compression Loads

There are no currently approved suppressive shielding applications which employ reinforced concrete columns. As discussed previously for steel, such columns properly proportioned for the outward blast loads would almost assuredly be satisfactory for the normal vertical static service loads and rebound loads. The adequacy is readily verified by procedures presented in Ref. 5-7.

5.3.4 Cylinders

The cylindrical pressure vessel is not normally used for suppressive shielding applications. Although Shield Groups 1, 2 and 3 are cylindrical in shape, they are an assemblage of beam, ring and plate elements. However, the equations for design or analysis of cylindrical pressure vessels can be applied to pipelines, ductwork, spheres and steel hoops and are, therefore, of interest for suppressive shield applications. It is also possible that the cylinder might be adapted to some future suppressive shielding application. In suppressive shield applications, the cylinder would be subjected to large internal dynamic pressure loads and zero or near zero external loads. The structural material will be responding primarily in tension, and buckling and moment loads, which are so important for cylinders subjected to external load, will be insignificant except in the vicinity of the end caps. Stresses in the vicinity of the juncture between the end caps and the cylinder walls are a function of the relative stiffnesses of these elements. Their prediction is complex and cannot be treated here.

The cylinder shown in Fig. 5-4 can be considered thin walled, if its wall thickness is equal to or less than one-tenth the internal radius. The average stress calculated for the wall thickness is a good approximation of the maximum stress in the wall. The force P acting on the end cap and base plate is the product of the internal pressure and the internal cross sectional area

$$P = p \times \pi R^2$$

where p is the internal pressure and R is the internal radius. The longitudinal stress in the cylinder wall depicted in Fig. 5-4 can be found from

$$\text{(Thin Wall)} \qquad \sigma_{\ell} = P/A_{wall} = \frac{pR}{2t} \qquad (5\text{-}17)$$

The radius to the mid section of the wall of thickness t can be taken equal to the internal radius with little error for thin wall cylinders.

The total force P on the base plate is the same as on the hemispherical cap. This force could be divided equally among the bolts that are shown or distributed around the circumference for welding when determining the end cap connection requirements.

The cylinder hoop forces are also depicted in Fig. 5-4. For the unit width strip shown,

$$F = 2Rp = 2H = 2\sigma_h t$$

and the hoop stress

$$\text{(Thin Wall)} \qquad \sigma_h = \frac{pR}{t} \qquad (5\text{-}18)$$

with all terms as previously defined.

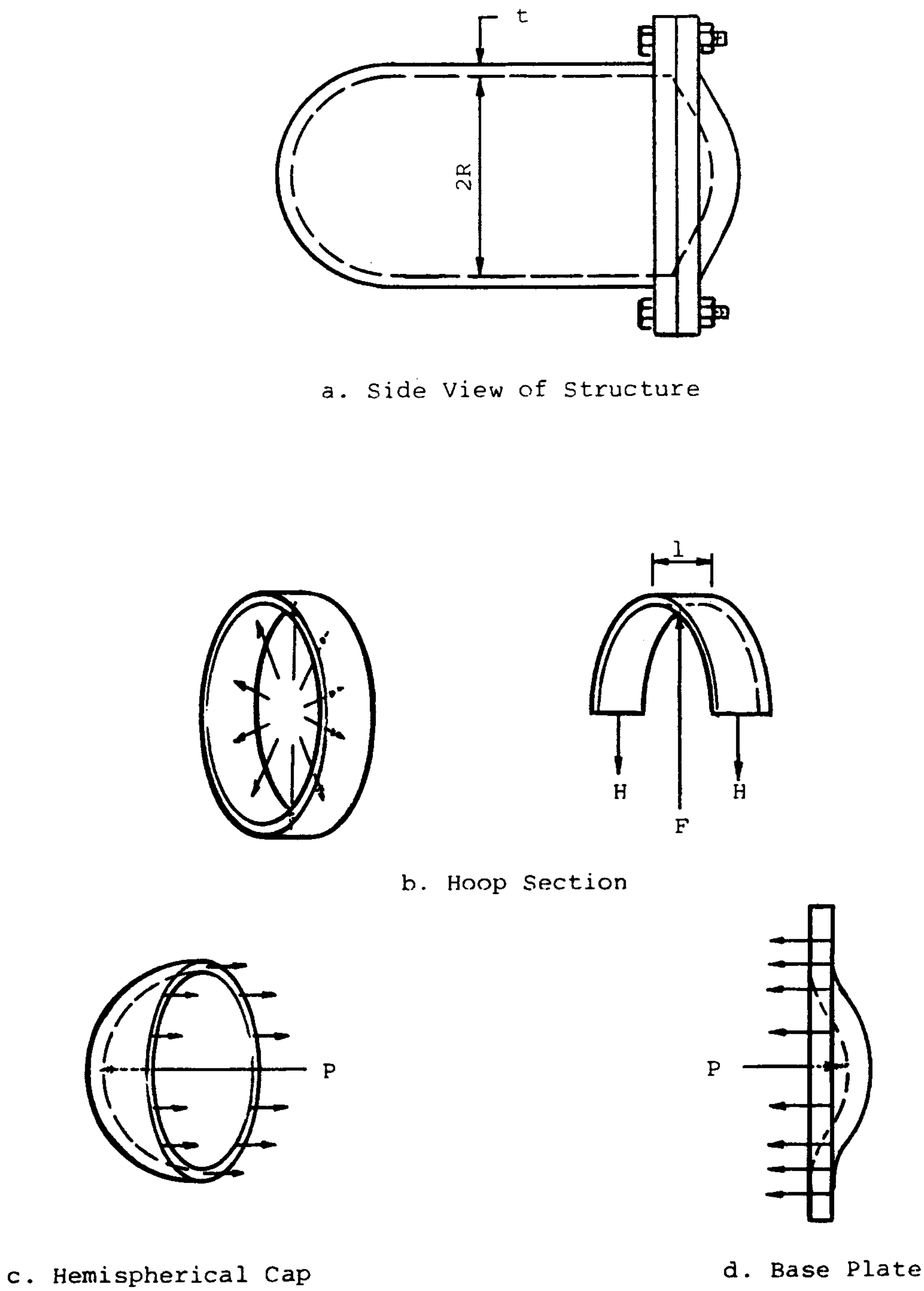


Figure 5-4. Cylindrical Structural Configuration

In cases where the cylinder wall thickness exceeds one-tenth of the internal radius, expressions for stress in thick wall cylinders should be used. The maximum hoop stress in a thick wall cylinder subjected to internal pressure only expressed as a ratio of the thin wall stress and a function of the outside and inside radii is shown in Fig. 5-5. Expressions from Ref. 5-10 for the maximum hoop, radial and longitudinal stresses in thick walled cylinders subjected to internal pressure only are summarized below.

$$\text{(Thick Wall)} \qquad \sigma_h = \frac{p(R_o^2 + R_i^2)}{R_o^2 - R_i^2} \tag{5-19}$$

$$\text{(Thick Wall)} \qquad \sigma_r = p \tag{5-20}$$

$$\text{(Thick Wall)} \qquad \sigma_\ell = \frac{pR_i^2}{R_o^2 - R_i^2} \tag{5-21}$$

where R_i is the inside radius, R_o is the outside radius of the thick wall cylinder and the other terms are as previously defined.

The stresses given by Eqs. 5-19 and 5-20 are maximum values and occur at the inside wall of the cylinder. The maximum value that a radial stress may attain is equal to the internal pressure. The hoop stress is normally larger than the radial stress for the conditions of interest in suppressive shielding.

The longitudinal stress, Eq. 5-21, can be assumed to be uniformly distributed on any transverse wall section which is not close to a capped end. Near a capped end, the influence of the cap will cause nonuniformity in the stress distribution. It will usually be found that values of σ_ℓ are small relative to those of σ_h and σ_r.

Equations 5-17 through 5-21 can be used for design or analysis of suppressive shields by taking the allowable stress equal to f_{dy}.

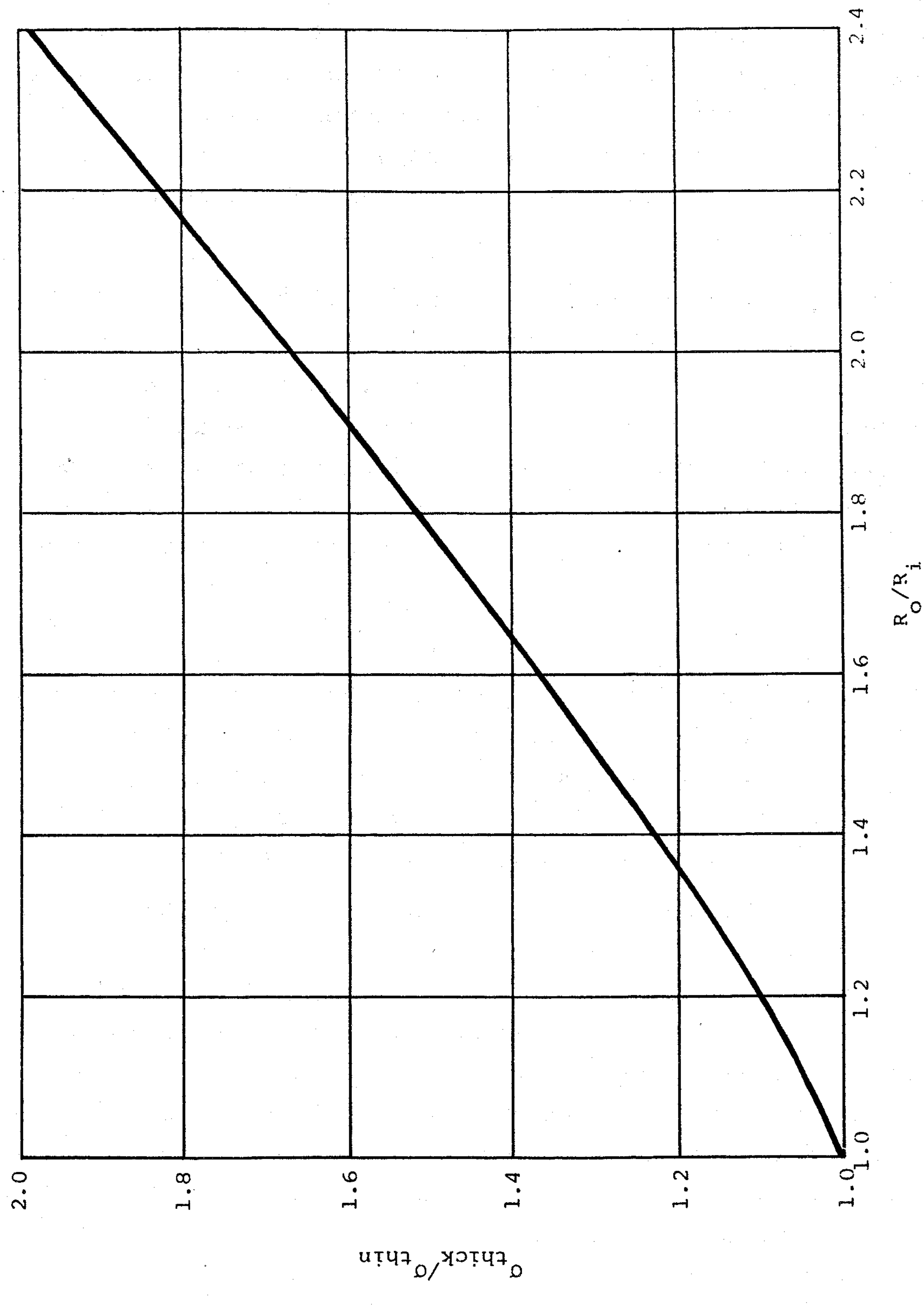


Figure 5-5. Hoop Stress in Thick Wall Cylinder Subjected to Internal Pressure Only

5.3.5 Spheres

Spherical chambers are used for some suppressive shield applications where the fragment hazards are minimal. Equations 5-17 and 5-21 can be used for calculating the maximum stresses in thin and thick wall spheres, respectively, subjected to an internal pressure only. The maximum static resistance of a spherical chamber would be obtained by taking the calculated stress equal to f_{dy}.

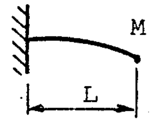
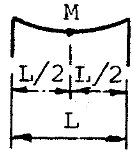
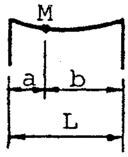
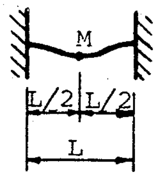
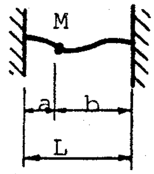
5.3.6 Natural Frequencies of Common Systems

Calculating natural frequencies is one of the important steps in the analysis of most systems. The expressions given in this section can be used to calculate the circular natural frequencies or period of vibration of various types of structural elements which remain elastic. The circular natural frequency and period of vibration of an element are related by

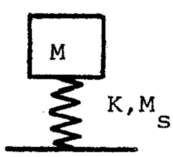
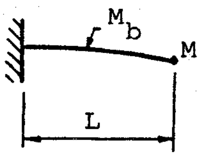
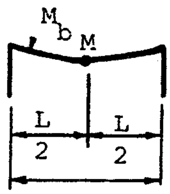
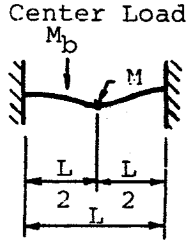
$$T_N = \frac{2\pi}{\omega_N} \tag{5-22}$$

where T_N is the period of vibration of the element in seconds and ω_N is the circular natural frequency in radians per second. If the period of vibration and load duration are known, the equations and charts of paragraph 5.5 and Appendix B can be used to obtain the maximum response of the system.

Figure 5-6 presents solutions for the circular natural frequency of various types of beams or one-way slabs. In Fig. 5-6(a), the mass of the beam is assumed to be very small compared to that of the supported load. The solutions given in Fig. 5-6(b) include consideration of both the mass of the beam and the supported mass. They do not include consideration of the stiffness added by attached plates or boxes. The importance of added stiffness depends on how much is added and over what portion of the span it extends. No general solution can be given here.

Fixed-Free End Load	Hinged-Hinged Center Load	Hinged-Hinged Off-Center Load	Fixed-Fixed Center Load	Fixed-Fixed Off-Center Load
				
$\omega_N = \sqrt{\frac{3EI}{ML^3}}$	$\omega_N = 4\sqrt{\frac{3EI}{ML^3}}$	$\omega_N = \frac{1}{ab}\sqrt{\frac{3EIL}{M}}$	$\omega_N = 8\sqrt{\frac{3EI}{ML^3}}$	$\omega_N = \frac{1}{ab}\sqrt{\frac{3EIL^3}{Mab}}$

(a) Massless Beams with Concentrated Mass Loads

Mass-Helical Spring	Fixed-Free End-Load	Hinged-Hinged Center Load	Fixed-Fixed Center Load
			
$\omega_N = \sqrt{\frac{K}{M + \frac{M_s}{3}}}$	$\omega_N = \sqrt{\frac{3EI}{L^3 (M+0.23M_b)}}$	$\omega_N = \sqrt{\frac{48EI}{L^3 (M+0.5M_b)}}$	$\omega_N = 14\sqrt{\frac{EI}{L^3 (M+0.375M_b)}}$

(b) Massive Springs (Beams) with Concentrated Mass Loads

M = Mass of Load, lb-sec^2/in

M_s (M_b) = Total Mass of Spring (Beam), lb-sec^2/in

K = Stiffness of Spring lb/in

L = Length of Beam, inches

I = Area Moment of inertia of Beam Cross Section, in^4

E = Young's Modulus, lb/in^2

ω_N = Natural Frequency, rad/sec

Figure 5-6. Natural Frequencies of Beam Elements with Concentrated Masses (Ref. 5-11)

Figure 5-7 presents solutions for the natural frequencies of circular and square slabs with various edge conditions. Figure 5-8 presents solutions for the natural frequencies of beams or one-way slabs with uniformly distributed mass and various support conditions.

The solutions presented in Figs. 5-6 through 5-8 are for the lowest (fundamental) mode of vibration. They do not include consideration of the effects of rotary motion and shearing forces on natural frequencies. These effects are small except for beams with small span to depth ratios, i.e., short, deep beams, or beams vibrating in higher mode shapes. Reference 5-11 presents guidance regarding adjustment of natural frequencies in those cases where these effects might be considered important.

Some cylindrical suppressive shields are strengthened with circumferential steel hoops. Under the radial loads imposed by the longitudinal beam columns they support, these steel hoops will respond in the extensional mode (all segments move radially together - in or out). The frequency of vibration of a steel hoop in this mode is given by

$$\omega_N = \sqrt{\frac{EA}{mR^2}} \qquad (5\text{-}23)$$

where

A = cross section area of beam, in^2

m = mass per unit length of beam, $lb\text{-}sec^2/in^2$

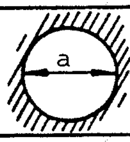
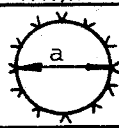
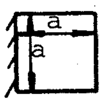
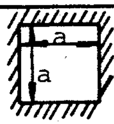
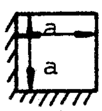
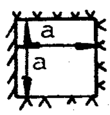
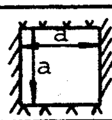
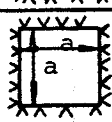
R = radius to center of beam, inches

The fundamental mode of vibration of a sphere would consist of simultaneous radial motion of all points on its surface. From Ref. 5-12, the natural period of vibration in the fundamental mode is given by

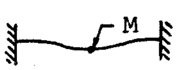
$$T_N = 2\pi\sqrt{\frac{\rho a^2(1-\nu)}{2E}} \qquad (5\text{-}24)$$

$$\omega_N = B\sqrt{\frac{Et^2}{\rho a^4 (1-\nu^2)}} \text{ rad/sec}$$

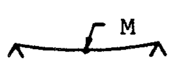
E = Young's Modulus, lb/in^2
t = Thickness of Plate, inches
ρ = Mass Density, $lb\text{-}sec^2/in^4$
a = Diameter of Circular Plate or Side of Square Plate, inches
ν = Poisson's Ratio

Shape of Plate	Diagram	Edge Conditions	Value of B
CIRCULAR		Clamped at Edge	11.84
CIRCULAR		Simply Supported at Edge	5.90
SQUARE		One Edge Clamped-Three Edges Free	1.01
SQUARE		All Edges Clamped	10.40
SQUARE		Two Edges Clamped-Two Edges Free	2.01
SQUARE		One Edge Clamped-Three Edges Simply Supported	6.83
SQUARE		Two Edges Clamped-Two Edges Simply Supported	8.37
SQUARE		All Edges Simply Supported	5.70

Massless Circular Plate with Concentrated Center Mass

Clamped Edges 

$$\omega_N = 4.09\sqrt{\frac{Et^3}{Ma^2(1-\nu^2)}}$$

Simply Supported Edges 

$$\omega_N = 4.09\sqrt{\frac{Et^3}{Ma^2(1-\nu)(3+\nu)}}$$

Figure 5-7. Fundamental Frequencies of Thin Flat Plates of Uniform Thickness (Ref. 5-11)

E = Young's Modulus, lb/in^2
I = Area Moment of inertia of Beam Cross Section, in^4
L = Length of Beam, inches
m = Mass Per Unit Length of Beam, $lb\text{-}sec^2/in^2$

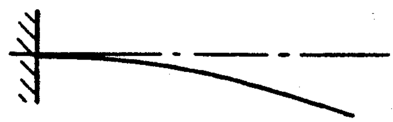
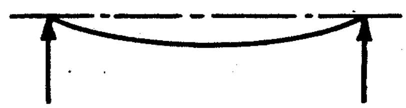
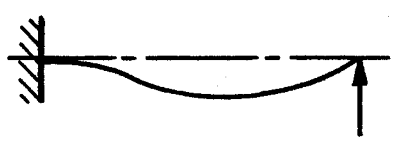
Fixed-Free (Cantilever)		$\omega_N = \frac{3.52}{L^2}\sqrt{\frac{EI}{m}}$
Hinged-Hinged (Simple)		$\omega_N = \frac{9.87}{L^2}\sqrt{\frac{EI}{m}}$
Fixed-Fixed (Built-in)		$\omega_N = \frac{22.4}{L^2}\sqrt{\frac{EI}{m}}$
Fixed-Hinged		$\omega_N = \frac{15.4}{L^2}\sqrt{\frac{EI}{m}}$

Figure 5-8. Natural Frequencies of Beams of Uniform Section and Uniformly Distributed Mass (Ref. 5-11)

where

a = radius of sphere, inches

ρ = mass density of sphere material, lb-sec^2/in^4

ν = Poisson's ratio for sphere material

E = modulus of elasticity for sphere material, lb/in^2

5.4 EQUIVALENT SINGLE DEGREE OF FREEDOM SYSTEMS

5.4.1 General

A rigorous dynamic analysis is feasible only for relatively simple structures where the loading and resistance functions can be expressed in simple mathematical terms. Although numerical analysis techniques are much more flexible, they also become tedious for more than a few degrees of freedom. Most real structures with distributed mass theoretically have an infinite number of degrees of freedom. For practical design purposes, it is necessary to develop approximate methods which allow rapid analysis of complex structures with reasonable accuracy. Fortunately, it is possible to reduce many common structural elements to an equivalent single degree of freedom system which can then be analyzed with accuracy sufficient for most engineering purposes. In view of the uncertainties in loads and material properties encountered in suppressive shield design, more complex analytical techniques are often not justified. The method used herein for reducing distributed mass systems to equivalent single degree of freedom systems is taken from Ref. 5-13.

Figure 5-9 shows a fixed end beam with a single degree of freedom replacement system. In order to define the equivalent single degree of freedom system, it is necessary to to determine the parameters $F_{eq}(t)$, M_{eq}, K_{eq} and X_{eq}. The usual approach is to define the system as one in which the equivalent displacement, velocity and acceleration are equal to that at some significant point in the actual system, e.g., the midspan of a beam. Stresses and forces in the equivalent system are not directly equivalent to those in the real system, but, if the deflections are known, the stresses in the real system can be calculated. It is also necessary to define equivalent resistance and forcing functions. The equivalent forcing function should have the same time dependence as the real load.

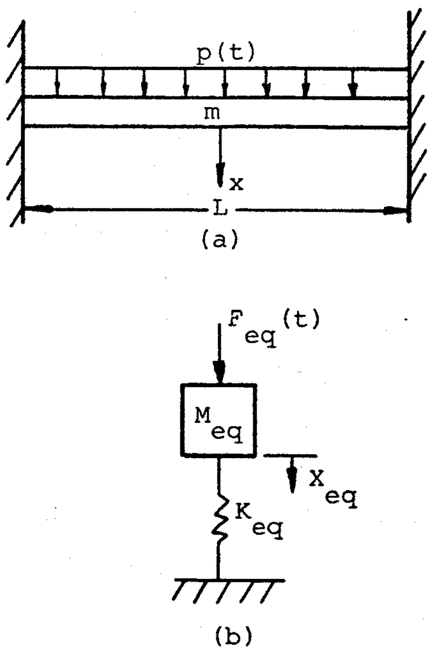


Figure 5-9. Equivalent Single Degree of Freedom System

The constants of the equivalent system are evaluated on the basis of an assumed deformed shape of the actual structure. This shape is usually taken as that resulting from the static application of the dynamic loads. This approach is not quite the same as that of using the first mode shape, but it yields more accurate results for many systems, especially for stress computations. These deflected shapes are more easily determined and described by simple mathematical functions than are mode shapes.

5.4.2 Transformation Factors for Beams and Slabs

It is convenient to develop transformation factors which convert the real system into the equivalent system. When the load, mass, resistance and stiffness of the real structure are multiplied by the corresponding transformation factors, these parameters are obtained for the equivalent single degree of freedom system. The mass transformation factor is defined to be

$$K_M = \frac{M_{eq}}{M_t} \tag{5-25}$$

where

M_t = total mass of the real structure

M_{eq} = mass of the equivalent single degree of freedom system

The load transformation factor is defined to be

$$K_L = \frac{F_{eq}}{F_t} \tag{5-26}$$

where

F_t = total force on the real structure

F_{eq} = force on the equivalent single degree of freedom system

Since the maximum resistance is the total load having the given distribution which the structure can support statically and the stiffness is equal to the total load of the same distribution required to cause a unit displacement at the significant point, it follows that the resistance factor, K_R, must always equal the load factor, K_L. Then

$$K_R = \frac{R_{meq}}{R_m} = K_L \tag{5-27}$$

and

$$K_R = \frac{K_{eq}}{K} = K_L \tag{5-28}$$

where R_m and K are the actual and R_{meq} and K_{eq} are the equivalent resistances and spring constants, respectively.

Transformation factors have been worked out for a number of common types of structural elements and support conditions. Tables 5-2 and 5-3 give factors for beams and one-way slabs, Tables 5-4 through 5-7 give factors for two-way slabs. Table 5-8 presents factors for circular slabs. The tables also include a load-mass factor which is defined to be the ratio of the mass and load-factors, i.e.,

$$K_{LM} = \frac{K_M}{K_L} \tag{5-29}$$

The ratio can be used to define the equations of motion for the equivalent system

$$K_{LM}M_t\ddot{X} + KX = F_t(t) \qquad \text{(elastic region)} \tag{5-30}$$

$$K_{LM}M_t\ddot{X} + R_m = F_t(t) \qquad \text{(plastic region)} \tag{5-31}$$

The natural frequency of both the real and idealized systems is

$$\omega_N = \left[\frac{K_{eq}}{M_{eq}}\right]^{1/2} = \left[\frac{K}{K_{LM}M_t}\right]^{1/2} \tag{5-32}$$

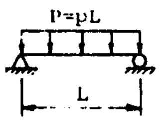
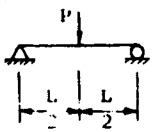
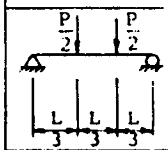
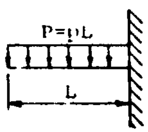
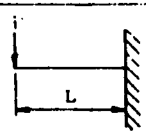
and the natural period is

$$T_n = 2\pi\left[\frac{K_{LM}M_t}{K}\right]^{1/2} \tag{5-33}$$

The maximum resistances and spring stiffnesses presented in Table 5-2 are those for the real system and are the conventional expressions for these quantities. They are given in terms of the total load on the system and, when multiplied by the load factor, they become the corresponding

Table 5-2

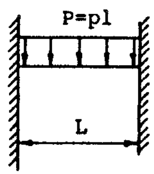
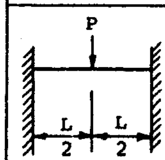
TRANSFORMATION FACTORS FOR BEAMS AND ONE-WAY SLABS (Refs. 5-6, 5-9, 5-1

Loading Diagram	Strain Range	Load Factor K_L	Mass Factor K_M		Load-Mass Factor K_{LM}		Maximum Resistance R_m	Spring Constant K	Dynamic Reaction V
			Concen-trated Mass*	Uniform Mass	Concen-trated Mass*	Uniform Mass			
	Elastic	0.64		0.50		0.78	$8M_p/L$	$384EI/5L^3$	$0.39R + 0.11F$
	Plastic	0.50		0.33		0.66	$8M_p/L$	0	$0.38R_m + 0.12F$
	Elastic	1.0	1.0	0.49	1.0	0.49	$4M_p/L$	$48EI/L^3$	$0.78R - 0.28F$
	Plastic	1.0	1.0	0.33	1.0	0.33	$4M_p/L$	0	$0.75R_m - 0.25F$
	Elastic	0.87	0.76	0.52	0.87	0.60	$6M_p/L$	$56.4EI/L^3$	$0.62R - 0.12F$
	Plastic	1.0	1.0	0.56	1.0	0.56	$6M_p/L$	0	**$0.52R_m - 0.02F$**
	Elastic	0.4		0.26		0.65	$2M_{Ps}/L$	$8EI/L^3$	$0.69R + 0.31F$
	Plastic	0.5		0.33		0.66	$2M_{Ps}/L$	0	$0.75R_m + 0.25F$
	Elastic	1.0	1.0	0.24	1.0	0.24	M_{Ps}/L	$3EI/L^3$	$1.36R - 0.36F$
	Plastic	1.0	1.0	0.33	1.0	0.33	M_{Ps}/L	0	$1.5R_m - 0.5F$

*Equal parts of the concentrated mass are lumped at each concentrated load.

Table 5-3

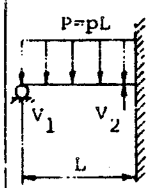
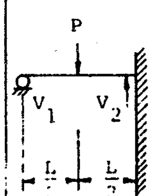
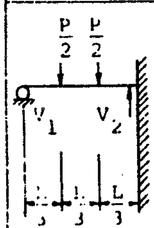
TRANSFORMATION FACTORS FOR BEAMS AND ONE-WAY SLABS (Ref. 5-13)

Loading Diagram	Strain Range	Load Factor K_L	Mass Factor K_M		Load-Mass Factor K_{LM}		Maximum Resistance R_m	Spring Constant K	Effective Spring Constant K_E	Dynamic Reaction V
			Concentrated Mass*	Uniform Mass	Concentrated Mass*	Uniform Mass				
	Elastic	0.53	....	0.41	....	0.77	$12M_{Ps}/L$	$384EI/L^3$	....	0.36R + 0.14F
	Elasto-Plastic	0.64	....	0.50	....	0.78	$\frac{8}{L}(M_{Ps} + M_{Pm})$	$384EI/5L^3$	$307EI/L^3$	0.39R + 0.11F
	Plastic	0.50	....	0.33	....	0.66	$\frac{8}{L}(M_{Ps} + M_{Pm})$	0	....	$0.38R_m + 0.12F$
	Elastic	1.0	1.0	0.37	1.0	0.37	$\frac{4}{L}(M_{Ps} + M_{Pm})$	$192EI/L^3$	....	0.71R - 0.21F
	Plastic	1.0	1.0	0.33	1.0	0.33	$\frac{4}{L}(M_{Ps} + M_{Pm})$	0	....	$0.75R_m - 0.25F$

*Concentrated mass is lumped at the concentrated load.

Table 5-3 (concluded)

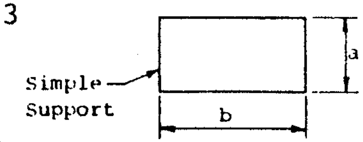
TRANSFORMATION FACTORS FOR BEAMS AND ONE-WAY SLABS (Ref. 5-6, 5-13)

Loading Diagram	Strain Range	Load Factor K_L	Mass Factor K_M		Load-Mass Factor K_{LM}		Maximum Resistance R_m	Spring Constant K	Effective Spring Constant K_E	Dynamic Reaction V
			Concentrated Mass*	Uniform Mass	Concentrated Mass*	Uniform Mass				
	Elastic	0.58	....	0.45	....	0.78	$8M_{Ps}/L$	$185EI/L^3$	....	$V_1 = 0.26R + 0.12F$ $V_2 = 0.43R + 0.19F$
	Elasto-Plastic	0.64	....	0.50	....	0.78	$\frac{4}{L}(M_{Ps} + 2M_{Pm})$	$384EI/5L^3$	$160EI/L^3$	$V_1 = 0.39R + 0.11F - M_{ps}/L$ $V_2 = 0.39R + 0.11F + M_{ps}/L$
	Plastic	0.50	....	0.33	....	0.66	$\frac{4}{L}(M_{Ps} + 2M_{Pm})$	0	....	$V_1 = 0.38R_m + 0.12F - M_{ps}/L$ $V_2 = 0.38R_m + 0.12F + M_{ps}/L$
	Elastic	1.0	1.0	0.43	1.0	0.43	$16M_{Ps}/3L$	$107EI/L^3$	....	$V_1 = 0.25R + 0.07F$ $V_2 = 0.54R + 0.14F$
	Elasto-Plastic	1.0	1.0	0.49	1.0	0.49	$\frac{2}{L}(M_{Ps} + 2M_{Pm})$	$48EI/L^3$	$160EI/L^3$	$V_1 = 0.78R - 0.28F - M_{ps}/L$ $V_2 = 0.78R - 0.28F + M_{ps}/L$
	Plastic	1.0	1.0	0.33	1.0	0.33	$\frac{2}{L}(M_{Ps} + 2M_{Pm})$	0	....	$V_1 = 0.75R_m - 0.25F - M_{ps}/L$ $V_2 = 0.75R_m - 0.25F + M_{ps}/L$
	Elastic	0.81	0.67	0.45	0.83	0.55	$6M_{Ps}/L$	$132EI/L^3$	....	$V_1 = 0.17R + 0.17F$ $V_2 = 0.33R + 0.33F$
	Elasto-Plastic	0.87	0.76	0.52	0.87	0.60	$\frac{2}{L}(M_{Ps} + 3M_{Pm})$	$56EI/L^3$	$122EI/L^3$	$V_1 = 0.62R - 0.12F - M_{ps}/L$ $V_2 = 0.62R - 0.12F + M_{ps}/L$
	Plastic	1.0	1.0	0.56	1.0	0.56	$\frac{2}{L}(M_{Ps} + 3M_{Pm})$	....	....	$V_1 = 0.52R_m - 0.02F - M_{ps}/L$ $V_2 = 0.52R_m - 0.02F + M_{ps}/L$

*Equal parts of the concentrated mass are lumped at each concentrated load.

Table 5-4

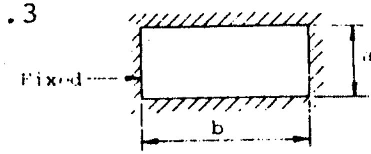
TRANSFORMATION FACTORS FOR TWO-WAY SLABS SIMPLE SUPPORTS, UNIFORM LOAD (Ref. 5-6, 5-13)
For Poissons Ratio = 0.3



Strain Range	a/b	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}	Maximum Resistance	Spring Constant K	Dynamic Reactions V_A	Dynamic Reactions V_B
Elastic	1.0	0.45	0.31	0.68	$\frac{12}{a}(M_{Pfa} + M_{Pfb})$	$252 EI_a/a^2$	$0.07F + 0.18R$	$0.07F + 0.18R$
	0.9	0.47	0.33	0.70	$\frac{1}{a}(12M_{Pfa} + 11M_{Pfb})$	$230 EI_a/a^2$	$0.06F + 0.16R$	$0.08F + 0.20R$
	0.8	0.49	0.35	0.71	$\frac{1}{a}(12M_{Pfa} + 10.3M_{Pfb})$	$212 EI_a/a^2$	$0.06F + 0.14R$	$0.08F + 0.22R$
	0.7	0.51	0.37	0.73	$\frac{1}{a}(12M_{Pfa} + 9.8M_{Pfb})$	$201 EI_a/a^2$	$0.05F + 0.13R$	$0.08F + 0.24R$
	0.6	0.53	0.39	0.74	$\frac{1}{a}(12M_{Pfa} + 9.3M_{Pfb})$	$197 EI_a/a^2$	$0.04F + 0.11R$	$0.09F + 0.26R$
	0.5	0.55	0.41	0.75	$\frac{1}{a}(12M_{Pfa} + 9.0M_{Pfb})$	$201 EI_a/a^2$	$0.04F + 0.09R$	$0.09F + 0.28R$
Plastic	1.0	0.33	0.17	0.51	$\frac{12}{a}(M_{Pfa} + M_{Pfb})$	0	$0.09F + 0.16R_m$	$0.09F + 0.16R_m$
	0.9	0.35	0.18	0.51	$\frac{1}{a}(12M_{Pfa} + 11M_{Pfb})$	0	$0.08F + 0.15R_m$	$0.09F + 0.18R_m$
	0.8	0.37	0.20	0.54	$\frac{1}{a}(12M_{Pfa} + 10.3M_{Pfb})$	0	$0.07F + 0.13R_m$	$0.10F + 0.20R_m$
	0.7	0.38	0.22	0.58	$\frac{1}{a}(12M_{Pfa} + 9.8M_{Pfb})$	0	$0.06F + 0.12R_m$	$0.10F + 0.22R_m$
	0.6	0.40	0.23	0.58	$\frac{1}{a}(12M_{Pfa} + 9.3M_{Pfb})$	0	$0.05F + 0.10R_m$	$0.10F + 0.25R_m$
	0.5	0.42	0.25	0.59	$\frac{1}{a}(12M_{Pfa} + 9.0M_{Pfb})$	0	$0.04F + 0.08R_m$	$0.11F + 0.27R_m$

Table 5-5

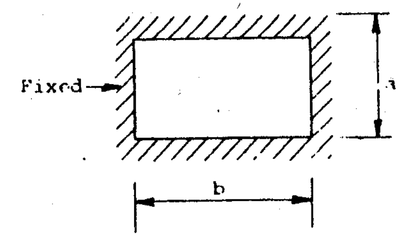
TRANSFORMATION FACTORS FOR TWO-WAY SLABS: FIXED SUPPORTS - UNIFORM LOAD (Ref. 5-6, 5-13)
For Poissons Ratio = 0.3



Strain Range	a/b	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}	Maximum Resistance	Spring Constant K	Dynamic Reactions V_A	Dynamic Reactions V_B
Elastic	1.0	0.33	0.21	0.63	$29.2 M^{o}_{Psb}$	$810\ EI_a/a^2$	0.10F + 0.15R	0.10F + 0.15R
	0.9	0.34	0.23	0.68	$27.4 M^{o}_{Psb}$	$742\ EI_a/a^2$	0.09F + 0.14R	0.10F + 0.17R
	0.8	0.36	0.25	0.69	$26.4 M^{o}_{Psb}$	$705\ EI_a/a^2$	0.08F + 0.12R	0.11F + 0.19R
	0.7	0.38	0.27	0.71	$26.2 M^{o}_{Psb}$	$692\ EI_a/a^2$	0.07F + 0.11R	0.11F + 0.21R
	0.6	0.41	0.29	0.71	$27.3 M^{o}_{Psb}$	$724\ EI_a/a^2$	0.06F + 0.09R	0.12F + 0.23R
	0.5	0.43	0.31	0.72	$30.2 M^{o}_{Psb}$	$806\ EI_a/a^2$	0.05F + 0.08R	0.12F + 0.25R
Elasto-Plastic	1.0	0.46	0.31	0.67	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 12(M_{Pfb} + M_{Psb})\right]$	$252\ EI_a/a^2$	0.07F + 0.18R	0.07F + 0.18R
	0.9	0.47	0.33	0.70	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 11(M_{Pfb} + M_{Psb})\right]$	$230\ EI_a/a^2$	0.06F + 0.16R	0.08F + 0.20R
	0.8	0.49	0.35	0.71	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 10.3(M_{Pfb} + M_{Psb})\right]$	$212\ EI_a/a^2$	0.06F + 0.14R	0.08F + 0.22R
	0.7	0.51	0.37	0.73	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.8(M_{Pfb} + M_{Psb})\right]$	$201\ EI_a/a^2$	0.05F + 0.13R	0.08F + 0.24R
	0.6	0.53	0.39	0.74	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.3(M_{Pfb} + M_{Psb})\right]$	$197\ EI_a/a^2$	0.04F + 0.11R	0.09F + 0.26R
	0.5	0.55	0.41	0.75	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.0(M_{Pfb} + M_{Psb})\right]$	$201\ EI_a/a^2$	0.04F + 0.09R	0.09F + 0.28R

Table 5-5 (concluded)

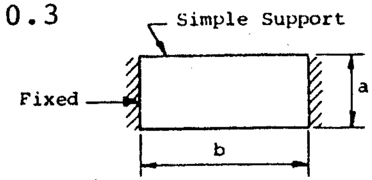
TRANSFORMATION FACTORS FOR TWO-WAY SLABS: FIXED SUPPORTS - UNIFORM LOAD (Ref. 5-13)
For Poissons Ratio = 0.3



Strain Range	a/b	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}	Maximum Resistance	Spring Constant K	Dynamic Reactions V_A	Dynamic Reactions V_B
Plastic	1.0	0.33	0.17	0.51	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 12(M_{Pfb} + M_{Psb})\right]$	0	$0.09F + 0.16R_m$	$0.09F + 0.16R_m$
	0.9	0.35	0.18	0.51	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 11(M_{Pfb} + M_{Psb})\right]$	0	$0.08F + 0.15R_m$	$0.09F + 0.18R_m$
	0.8	0.37	0.20	0.54	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 10.3(M_{Pfb} + M_{Psb})\right]$	0	$0.07F + 0.13R_m$	$0.10F + 0.20R_m$
	0.7	0.38	0.22	0.58	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.8(M_{Pfb} + M_{Psb})\right]$	0	$0.06F + 0.12R_m$	$0.10F + 0.22R_m$
	0.6	0.40	0.23	0.58	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.3(M_{Pfb} + M_{Psb})\right]$	0	$0.05F + 0.10R_m$	$0.10F + 0.25R_m$
	0.5	0.42	0.25	0.59	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.0(M_{Pfb} + M_{Psb})\right]$	0	$0.04F + 0.08R_m$	$0.11F + 0.27R_m$

Table 5-6

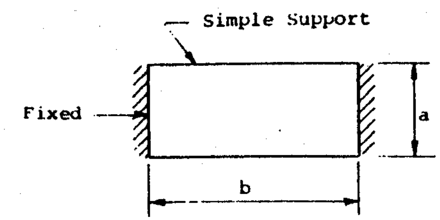
TRANSFORMATION FACTORS FOR TWO-WAY SLABS: SHORT EDGES FIXED - LONG EDGES SIMPLY SUPPORTED (Ref. 5-13) For Poissons Ratio = 0.3



Strain Range	a/b	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}	Maximum Resistance	Spring Constant K	Dynamic Reactions V_A	Dynamic Reactions V_B
Elastic	1.0	0.39	0.26	0.67	$20.4M^o_{Psa}$	$575EI_a/a^2$	0.09F + 0.16R	0.07F + 0.18R
	0.9	0.41	0.28	0.68	$10.2M^o_{Psa} + \frac{11}{a} M_{Pfb}$	$476EI_a/a^2$	0.08F + 0.14R	0.08F + 0.20R
	0.8	0.44	0.30	0.68	$10.2M^o_{Psa} + \frac{10.3}{a} M_{Pfb}$	$396EI_a/a^2$	0.08F + 0.12R	0.08F + 0.22R
	0.7	0.46	0.33	0.72	$9.3M^o_{Psa} + \frac{9.7}{a} M_{Pfb}$	$328EI_a/a^2$	0.07F + 0.11R	0.08F + 0.24R
	0.6	0.48	0.35	0.73	$8.5M^o_{Psa} + \frac{9.3}{a} M_{Pfb}$	$283EI_a/a^2$	0.06F + 0.09R	0.09F + 0.26R
	0.5	0.51	0.37	0.73	$7.4M^o_{Psa} + \frac{9.0}{a} M_{Pfb}$	$243EI_a/a^2$	0.05F + 0.08R	0.09F + 0.28R
Elasto-Plastic	1.0	0.46	0.31	0.67	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 12(M_{Pfb})\right]$	$271EI_a/a^2$	0.07F + 0.18R	0.07F + 0.18R
	0.9	0.47	0.33	0.70	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 11(M_{Pfb})\right]$	$248EI_a/a^2$	0.06F + 0.16R	0.08F + 0.20R
	0.8	0.49	0.35	0.71	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 10.3(M_{Pfb})\right]$	$228EI_a/a^2$	0.06F + 0.14R	0.08F + 0.22R
	0.7	0.51	0.37	0.72	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.7(M_{Pfb})\right]$	$216EI_a/a^2$	0.05F + 0.13R	0.08F + 0.24R
	0.6	0.53	0.37	0.70	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.3(M_{Pfb})\right]$	$212EI_a/a^2$	0.04F + 0.11R	0.09F + 0.26R
	0.5	0.55	0.41	0.74	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.0(M_{Pfb})\right]$	$216EI_a/a^2$	0.04F + 0.09R	0.09F + 0.28R

Table 5-6 (concluded)

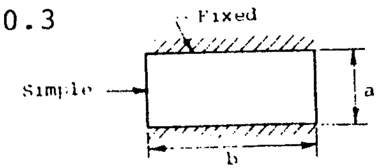
TRANSFORMATION FACTORS FOR TWO-WAY SLABS:
SHORT EDGES FIXED - LONG EDGES SIMPLY SUPPORTED (Ref. 5-13)
For Poissons Ratio = 0.3



Strain Range	a/b	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}	Maximum Resistance	Spring Constant K	Dynamic Reaction V_A	Dynamic Reaction V_B
Plastic	1.0	0.33	0.17	0.51	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 12M_{Pfb}\right]$	0	$0.09F + 0.16R_m$	$0.09F + 0.16R_m$
	0.9	0.35	0.18	0.51	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 11M_{Pfb}\right]$	0	$0.08F + 0.15R_m$	$0.09F + 0.18R_m$
	0.8	0.37	0.20	0.54	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 10.3M_{Pfb}\right]$	0	$0.07F + 0.13R_m$	$0.10F + 0.20R_m$
	0.7	0.38	0.22	0.58	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.7M_{Pfb}\right]$	0	$0.06F + 0.12R_m$	$0.10F + 0.22R_m$
	0.6	0.40	0.23	0.58	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.3M_{Pfb}\right]$	0	$0.05F + 0.10R_m$	$0.10F + 0.25R_m$
	0.5	0.42	0.25	0.59	$\frac{1}{a}\left[12(M_{Pfa} + M_{Psa}) + 9.0M_{Pfb}\right]$	0	$0.04F + 0.08R_m$	$0.11F + 0.27R_m$

Table 5-7

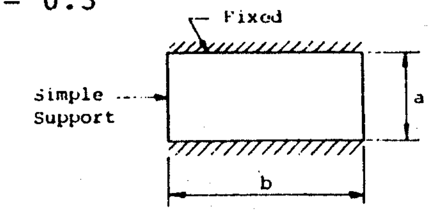
TRANSFORMATION FACTORS FOR TWO-WAY SLABS:
SHORT SIDES SIMPLY SUPPORTED - LONG SIDES FIXED (Ref. 5-13)
For Poissons Ratio = 0.3



Strain Range	a/b	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}	Maximum Resistance	Spring Constant K	Dynamic Reactions V_A	Dynamic Reactions V_B
Elastic	1.0	0.39	0.26	0.67	$20.4M^o_{Psb}$	$575EI_a/a^2$	0.07F + 0.18R	0.09F + 0.16R
	0.9	0.40	0.28	0.70	$19.5M^o_{Psb}$	$600EI_a/a^2$	0.06F + 0.16R	0.10F + 0.18R
	0.8	0.42	0.29	0.69	$19.5M^o_{Psb}$	$610EI_a/a^2$	0.06F + 0.14R	0.11F + 0.19R
	0.7	0.43	0.31	0.71	$20.2M^o_{Psb}$	$662EI_a/a^2$	0.05F + 0.13R	0.11F + 0.21R
	0.6	0.45	0.33	0.73	$21.2M^o_{Psb}$	$731EI_a/a^2$	0.04F + 0.11R	0.12F + 0.23R
	0.5	0.45	0.34	0.72	$22.2M^o_{Psb}$	$850EI_a/a^2$	0.04F + 0.09R	0.12F + 0.25R
Elasto-Plastic	1.0	0.46	0.31	0.67	$\frac{1}{a}\left[12M_{Pfa} + 12(M_{Psb} + M_{Pfb})\right]$	$271EI_a/a^2$	0.07F + 0.18R	0.07F + 0.18R
	0.9	0.47	0.33	0.70	$\frac{1}{a}\left[12M_{Pfa} + 11(M_{Psb} + M_{Pfb})\right]$	$248EI_a/a^2$	0.06F + 0.16R	0.08F + 0.20R
	0.8	0.49	0.35	0.71	$\frac{1}{a}\left[12M_{Pfa} + 10.3(M_{Psb} + M_{Pfb})\right]$	$228EI_a/a^2$	0.06F + 0.14R	0.08F + 0.22R
	0.7	0.51	0.37	0.73	$\frac{1}{a}\left[12M_{Pfa} + 9.8(M_{Psb} + M_{Pfb})\right]$	$216EI_a/a^2$	0.05F + 0.13R	0.08F + 0.24R
	0.6	0.53	0.39	0.74	$\frac{1}{a}\left[12M_{Pfa} + 9.3(M_{Psb} + M_{Pfb})\right]$	$212EI_a/a^2$	0.04F + 0.11R	0.09F + 0.26R
	0.5	0.55	0.41	0.74	$\frac{1}{a}\left[12M_{Pfa} + 9.0(M_{Psb} + M_{Pfb})\right]$	$216EI_a/a^2$	0.04F + 0.09R	0.09F + 0.28R

Table 5-7 (concluded)

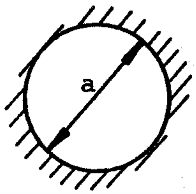
TRANSFORMATION FACTORS FOR TWO-WAY SLABS
SHORT SIDES SIMPLY SUPPORTED - LONG SIDES FIXED (Ref. 5-13)
For Poissons Ratio = 0.3



Strain Range	a, b	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}	Maximum Resistance	Spring Constant K	Dynamic Reactions V_A	Dynamic Reactions V_B
Plastic	1.0	0.33	0.17	0.51	$\frac{1}{a}\left[12M_{Pfa} + 12(M_{Psb} + M_{Pfb})\right]$	0	$0.09F + 0.16R_m$	$0.09F + 0.16R_m$
	0.9	0.35	0.18	0.51	$\frac{1}{a}\left[12M_{Pfa} + 11(M_{Psb} + M_{Pfb})\right]$	0	$0.08F + 0.15R_m$	$0.09F + 0.18R_m$
	0.8	0.37	0.20	0.54	$\frac{1}{a}\left[12M_{Pfa} + 10.3(M_{Psb} + M_{Pfb})\right]$	0	$0.07F + 0.13R_m$	$0.10R + 0.20R_m$
	0.7	0.38	0.22	0.58	$\frac{1}{a}\left[12M_{Pfa} + 9.8(M_{Psb} + M_{Pfb})\right]$	0	$0.06F + 0.12R_m$	$0.10F + 0.22R_m$
	0.6	0.40	0.23	0.58	$\frac{1}{a}\left[12M_{Pfa} + 9.3(M_{Psb} + M_{Pfb})\right]$	0	$0.05F + 0.10R_m$	$0.10F + 0.25R_m$
	0.5	0.42	0.25	0.59	$\frac{1}{a}\left[12M_{Pfa} + 9.0(M_{Psb} + M_{Pfb})\right]$	0	$0.04F + 0.08R_m$	$0.11F + 0.27R_m$

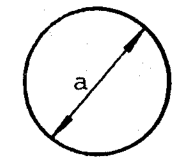
Table 5-8

TRANSFORMATION FACTORS FOR CIRCULAR SLABS
FOR POISSONS RATIO = 0.3 (Ref. 5-9)



Fixed Edges

a = Diameter of Slab



Simple Supports

Edge Condition	Strain Range	K_L	K_M	K_{LM}	Maximum Resistance	Spring Constant	Dynamic Reaction
Simple Supports	Elastic	0.46	0.30	0.65	$18.8M_{pc}$	$216EI/a^2$	0.28F + 0.72R
	Plastic	0.33	0.17	0.52	$18.8M_{pc}$	0	$0.36F + 0.64R_m$
Fixed Supports	Elastic	0.33	0.20	0.61	$25.1M_{ps}$	$880EI/a^2$	0.40F + 0.60R
	Elasto-Plastic	0.46	0.30	0.65	$18.8(M_{pc} + M_{ps})$	$216EI/a^2$	0.28F + 0.72R
	Plastic	0.33	0.17	0.52	$18.8(M_{pc} + M_{ps})$	0	$0.36F + 0.64R_m$

quantities for the equivalent system. Maximum resistances are expressed in terms of the fully plastic moment capacity, M_p, of the element and are based on the assumption that the member is proportioned so that a shear failure is prevented. Expressions for the fully plastic moment capacity of steel and reinforced concrete members are given in paragraph 5.3. The resistance-displacement function is a bilinear one similar to the elastic-plastic one shown in Fig. 5-1. Note that two mass factors are given for beams with concentrated loads. The concentrated mass factor is applied to those concentrated masses which occur at the point of application of the loads. The total equivalent mass would be the sum of the equivalent contributions from concentrated and distributed masses.

The maximum resistances and spring constants presented in Table 5-3 are for beams or one-way slabs with one end fixed and the other simply supported or with both ends fixed. In these cases, the element goes through three ranges of response since the fully plastic condition does not coincide with the formation of a plastic hinge at the supports. The resistance-displacement function for these elements is similar to that shown in Fig. 5-2. An exception is the fixed end beam with a concentrated load at midspan. For this case, the moments at midspan and the supports are equal and there is no elastic-plastic range. The maximum resistances given in Table 5-3 are those which occur at the upper limit of each range. In addition to the spring constant for each range, an effective spring constant covering all ranges is given. This effective spring constant allows the establishment of a bilinear resistance displacement function for use with the expressions given in paragraph 5.5. If plastic deformation is allowed (the normal case for most suppressive shield elements), the plastic K_{eq} is used. An equivalent elastic limit displacement can be obtained from

$$X_{eq} = \frac{R_{meq}}{K_{eq}} \tag{5-34}$$

As in the case of Table 5-2, the maximum resistance and stiffnesses are in terms of the total load on the element. They must be multiplied by the appropriate load factor to obtain corresponding quantities for the equivalent system. The mass factors in Table 5-3 are used in the same manner as those in Table 5-2.

Maximum resistances, spring stiffnesses and transformation factors for two-way rectangular slabs with various edge conditions are presented in Tables 5-4 through 5-7. These quantities are also obtained from assumed deflected shapes of the slabs. In the elastic region, the deflected shapes are based upon approximations to classical plate theory. In the plastic region, they are based upon yield line theory. The assumed yield lines are as shown in Fig. 5-10. As in the case of the beams and slabs with fixed ends, the resistance-displacement function for two-way slabs can be divided into three ranges; elastic, elastic-plastic, and plastic. Reference 5-13 neglects the elastic-plastic range for simply supported two-way slabs. The elastic range for simply supported two-way slabs is assumed to exist until the development of plastic moment along the assumed yield lines. The elastic range for fixed-end, two-way slabs is assumed to exist until the support moment along the long edge of rectangular slabs, or all edges of square slabs, reaches the plastic resistance value. This is the beginning of the elastic-plastic range which is assumed to hold up to the plastic range. The plastic range is initiated with the development of plastic moment along each of the assumed yield lines. For two-way slabs, simply supported on two opposite sides and fixed on the other two sides, the elastic range is assumed to exist until the development of plastic moments along the fixed edges. The elastic-plastic range is assumed to hold up to the development of full plastic moments along all assumed yield lines. The maximum resistances given

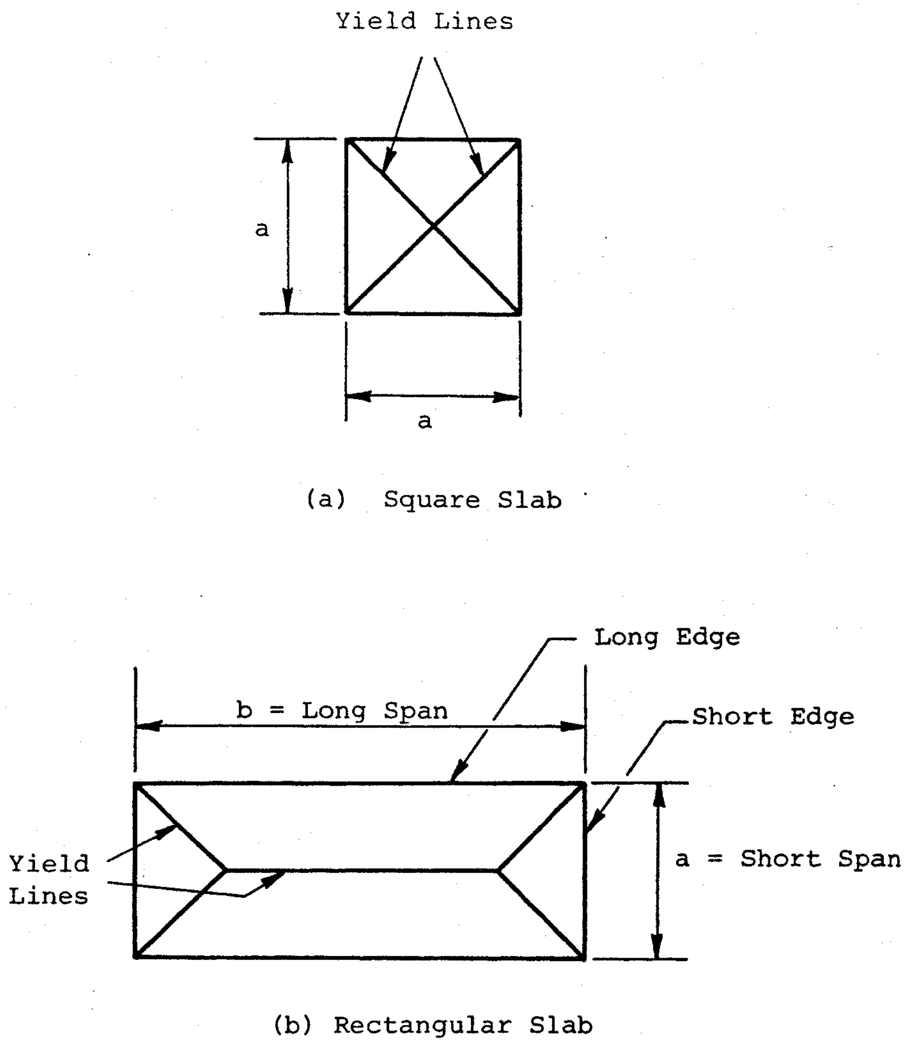


Figure 5-10. Assumed Yield Lines for Two-Way Slabs

in Tables 5-4 through 5-7 represent the upper limits of each range. Spring stiffnesses are given for each range of response. Maximum resistances and spring stiffnesses are given in terms of the total load on the slab and must be multiplied by the load factor to obtain the corresponding quantities for the equivalent single degree of freedom system. These transformed quantities can be used to construct a resistance displacement diagram similar to that shown in Fig. 5-2. An effective spring stiffness over the entire displacement range can be calculated using the procedure described earlier in paragraph 5.2. A reasonable approximation can be obtained by visual inspection of the tri-linear plot.

Table 5-8 presents transformation factors for circular slabs. These factors were taken from Ref. 5-9 and were derived using the procedures outlined in Ref. 5-13.

As in the case of beams and one-way slabs, the maximum resistances for two-way slabs assume that the slabs are proportioned so that they do not fail in shear.

The notation used in Tables 5-2 through 5-8 is as follows.

M_P = ultimate bending moment capacity

M_{Pfa} = total ultimate positive bending moment capacity along midspan section parallel to short edge, a

M_{Pfb} = total ultimate positive bending moment capacity along midspan section parallel to long edge, b

M_{Psa} = total ultimate negative moment capacity along short edge, a

M_{Psb} = total ultimate negative moment capacity along long edge, b

M°_{Psa} = ultimate negative bending moment capacity per unit width at center of edge a in direction of long span, b

M°_{Psb} = ultimate negative bending moment capacity per unit width at center of edge b in direction of short span, a

M_{Pc} = ultimate positive bending moment capacity per unit width at center of circular slab

M_{Ps} = ultimate negative bending moment capacity per unit width at edge of circular slab or ultimate bending moment capacity of beam at support

M_{Pm} = ultimate bending moment capacity of beam at mid-span

I = moment of inertia of beam or moment of inertia of unit width of slab

I_a = average of gross and cracked moment of inertia per unit width of concrete slabs (for short span in two-way slabs) or moment of inertia of plate per unit width

E = modulus of elasticity

V = dynamic reaction at ends of symmetric beams or simple cantilever

V_1 = dynamic reaction at hinged end of non-symmetric beams

V_2 = dynamic reaction at fixed end of non-symmetric beams

V_A = total dynamic reaction along one short edge

V_B = total dynamic reaction along one long edge

5.4.3 Dynamic Reactions

It is important to recognize that the dynamic reactions of the real structural element have no direct counterpart in the equivalent single degree of freedom system (Ref. 5-6). It is important to obtain some estimate of reactions since they are always related to the maximum shear in the element, and they are also necessary for the design of the supporting structures.

Expressions for the reactions may be obtained by considering the dynamic equilibrium of the complete element. The dynamic equilibrium of the element includes consideration of loads acting on the element and inertia forces which are assumed to be proportional to the deflected shape. By assuming a deflected shape, the reactions can be defined in terms of the loads acting on the element and its resistance. Tables 5-2 through 5-8 include factors for calculating the dynamic reactions of the various structural elements. The general form of the expression is

$$V = C_1F + C_2R \tag{5-35}$$

where

- V = the dynamic reaction at one end or edge of the element, except in the case of circular slabs where V represents the total reaction at the supports
- C_1, C_2 = coefficients obtained from the tables
- F = total force applied to the element
- R = resistance of the element

In most cases, both F and R are functions of time. In the elastic range, the maximum resistance occurs at maximum displacement, and the loading and resistance at the time of maximum displacement are used in Eq. 5-35 with the appropriate coefficients to obtain the dynamic reactions. In the plastic range, as the load F decreases with time, the maximum reactions occur when the displacement first reaches its yield value and the resistance is equal to R_m. For these cases, the time to reach yield displacement can be obtained by numerical integration of the equation of motion or by a slightly conservative method discussed in the next paragraph. The loading at this time and R_m are used in Eq. 5-35 to obtain the dynamic reactions.

In those cases where the quasi-static load does not decrease significantly within a period of time approximately equal to the period of the structure and the peak reflected pressure pulse whose time of duration is much less than the period is neglected, the procedure is simplified somewhat. For both the elastic and plastic regions, the loading contribution is taken equal to the peak quasi-static load and the resistance is taken equal to R_m. The latter approach can also be used to obtain a conservative estimate of reactions for a decaying pulse. For rapidly decaying loads, the results can be overly conservative. Note that since F and R are expressed in terms of the total load and resistance, the reactions obtained from Eq. 5-35 represent the total at the ends of the beam or the total for an edge of the slab.

5.5 DYNAMIC RESPONSE OF STRUCTURAL SYSTEMS

5.5.1 Introduction

Most real structures are very complex in their behavior even under static loads, and their response to dynamic loads includes additional complications due to various combinations of elastic and inelastic vibrational modes. The usual approach to determining the dynamic response of a structure or structural element to some specific loading is to first model or represent the structure as a system of finite structural elements and masses connected together at a discrete number of nodal points. If the force-displacement relationships are known for the individual elements, various methods of structural analysis can be used to study the behavior of the assembled structure. Most structures are made up of beams, girders, columns, slabs, plates and shells, with each of these elements having distributed mass and stiffness. If certain assumptions are made regarding stiffness of connections, lumping of masses, stiffnesses and applied loads, it is possible to replace these structures and structural elements with simpler equivalent systems. In general, the more complex the structure, the greater the number of individual elements required to accurately describe its response.

Methods of analysis for complex multiple degree of freedom systems are not considered in this handbook, except for those systems which can be represented by an equivalent single degree of freedom system. Approximate methods of analysis which reduce some common types of multiple degree of freedom systems to equivalent single degree of freedom systems were presented in paragraph 5.4. If the system is assumed to be vibrating in its fundamental mode only, its natural frequency can be computed using expressions from paragraph 5.3.6 and the system analyzed as a single degree of freedom system. Most suppressive shield structures consist of combinations of structural elements; however, it can be assumed in many cases that individual elements act independently of each other. For example, the peak response of a beam may be considered independent of the response of a girder which supports it. In this case, the beam and girder system can be analyzed independently as two uncoupled single degree of freedom systems. An approximate rule is that two such elements may be treated separately if their periods of vibration vary by a factor of 2 or more.

If the periods of vibration of the two elements are not sufficiently different, a multiple degree of freedom analysis should be made. The numerical integration technique which is applied to single degree of freedom systems in paragraph 5.5.3 can also be used for analysis of multiple degree of freedom systems. The calculations for systems with greater than two or three degrees of freedom become lengthy and tedious and should be programmed for high speed electronic computers. The elastic response of multiple degree of freedom systems is readily obtained using structural analysis programs such as STRUDL/DYNAL (MCAUTO), NASTRAN (NASA), STARDYNE (Mechanics Research, Inc.), and SAP IV (Univ. of Calif. at Berkeley). MARC-CDC (Control Data Corp.) and ANSYS (Swanson Analysis Systems, Inc.) are general purpose finite element programs for the nonlinear analysis of structures with large displacements.

5.5.2 Energy Methods

Energy and momentum considerations can be used to develop general solutions for single degree of freedom systems. Assuming an elastic-plastic resistance function such as that shown in Fig. 5-1, solutions can be obtained for load cases which approximate those generated within suppressive shields. The first corresponds to the situation where the load rises suddenly to its maximum value and remains constant for all displacements of the structural element. This is the step pulse or long duration loading shown in Fig. 5-11. The second is the case where all of the force is applied as an impulse before the structural element can displace appreciably.

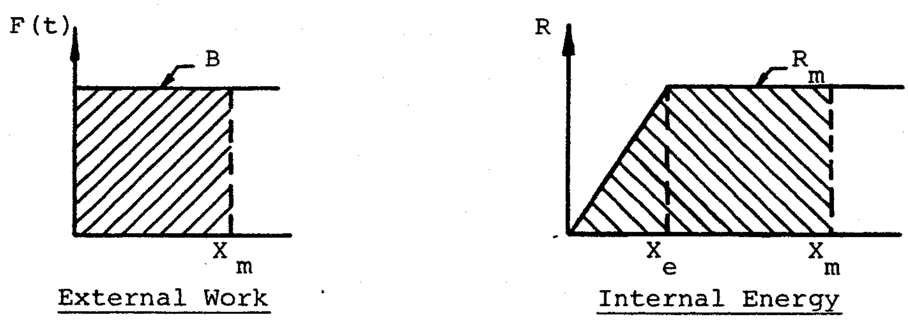


Figure 5-11. External Work and Internal Energy for Long Duration Load

In the first case, the external work done on the system is equal to the applied force times the displacement of its point of application in the direction of the force. At maximum displacement, the velocity of the mass is zero and the external work must be equal to the strain energy stored in the system. The external work and the internal energy are represented by the shaded areas of the diagrams shown in Fig. 5-11.

Equating the two areas

$$BX_m = R_m X_m - \frac{R_m X_e}{2} \tag{5-36}$$

the required maximum resistance is given by

$$R_m = B\left[\frac{2\mu}{2\mu-1}\right] \tag{5-37}$$

Rearranging terms in Eq. 5-37, the maximum displacement of the system is obtained from

$$\mu = \frac{X_m}{X_e} = \frac{1}{2\left[1 - \frac{B}{R_m}\right]} \tag{5-38}$$

Equation 5-37 is applicable only to those problems where $\mu \geq 1$. It should also be noted from Eq. 5-38 that R_m must be greater than the peak load, B. If $R_m \leq B$, the internal energy will never equal the external work done by the step pulse.

In the case of the impulsive load shown in Fig. 5-12, the total impulse applied to the system is equal to the area under the load-time function, i.e.,

$$i = \frac{Bt_o}{2} \tag{5-39}$$

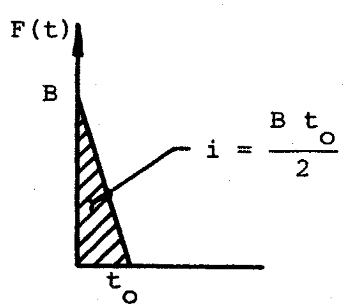


Figure 5-12. Impulsive Loading

Assuming the system is initially at rest, this impulse imparts an instantaneous velocity to the mass of

$$\dot{X} = \frac{i}{M} \tag{5-40}$$

and the kinetic energy of the mass is

$$K.E. = \frac{M(\dot{X})^2}{2} = \frac{i^2}{2M} \tag{5-41}$$

As in the case of the long duration load, this kinetic energy will be converted to strain energy at the time of maximum displacement of the system. Thus

$$\frac{1}{2M}\left[\frac{Bt_o}{2}\right]^2 = R_m X_m - \frac{R_m X_e}{2} \tag{5-42}$$

Substituting

$$M = \frac{K}{\omega_N^2} = \frac{R_m}{X_e}\left[\frac{T_N}{2\pi}\right]^2$$

and

$$\mu = \frac{X_m}{X_e}$$

into Eq. 5-42, the required maximum resistance is given by

$$R_m = B\left[\frac{\pi t_o}{T_N\sqrt{2\mu-1}}\right] \tag{5-43}$$

Making the substitution

$$i = \frac{Bt_o}{2}$$

and

$$T_N = \frac{2\pi}{\omega_N}$$

results in a more general form of Eq. 5-43.

$$R_m = \frac{i\omega_N}{\sqrt{2\mu-1}} \tag{5-44}$$

Maximum response of the system is obtained from

$$\mu = \frac{1}{2}\left[\left(\frac{B\pi t_o}{R_m T_N}\right)^2 + 1\right] \tag{5-45}$$

or

$$\mu = \frac{1}{2}\left[\left(\frac{i\omega_N}{R_m}\right)^2 + 1\right] \tag{5-46}$$

Equations 5-43 through 5-46 are also applicable only to problems where $\mu \geq 1$. In order for this condition to occur, the ratios

$$\frac{B\pi t_o}{R_m T_N} \quad \text{and} \quad \frac{i\omega_N}{R_m}$$

must be greater than or equal to 1. Equations 5-37 and 5-38 are most correct for larger values of t_o/T_N and Eqs. 5-43 through 5-46 for smaller values. Large and small have been somewhat arbitrarily defined to be ratios of 10 and 0.2 respectively. Reference 5-14 recommends the expression

$$\frac{B}{R_m} = \frac{T_N}{\pi t_o}\sqrt{2\mu - 1} + \frac{1 - \frac{1}{2\mu}}{1 + 0.7\,\frac{T_N}{t_o}} \tag{5-47}$$

as applicable over the whole range of possible values of t_o/T_N. Equation 5-47 is reported to be in error by less than 8.4 percent over a range of values of t_o from 0 to infinity and of μ from 1 to infinity. At **large** values of t_o/T_n, it reduces to Eq. 5-37; at small values of t_o/T_n, it becomes Eq. 5-43.

The methods of analyses presented up to this point can only be applied to loadings which can be adequately represented by a simple triangular or step function. In some instances, a multiple triangle approximation of the actual loading will yield more accurate results. Figure 5-13 shows a three-triangle approximation of a loading function. Two, four or n-triangle approximations are also possible. A reasonable approximation to the response of a single degree of freedom system to this loading can be obtained by treating each triangle as a partial load acting alone. For the three triangle approximation shown in Fig. 5-13, the relationship is

$$\frac{C_1 B/R_m}{F_1} + \frac{C_2 B/R_m}{F_2} + \frac{C_3 B/R_m}{F_3} = 1 \tag{5-48}$$

where F_1, F_2, and F_3 are the values of C_1B/R_m, C_2B/R_m and C_3B/R_m for given values of μ and ratios of duration of load to period t_1/T_N, t_2/T_N and t_3/T_N, respectively. Equations 5-37, 5-43, 5-44 or 5-47, as appropriate, can be used to obtain F_1, F_2 and F_3. Identical values of μ and T_N are used for each partial load computation. The general relationship is

$$\sum \frac{C_n B/R_m}{F_n} = 1 \tag{5-49}$$

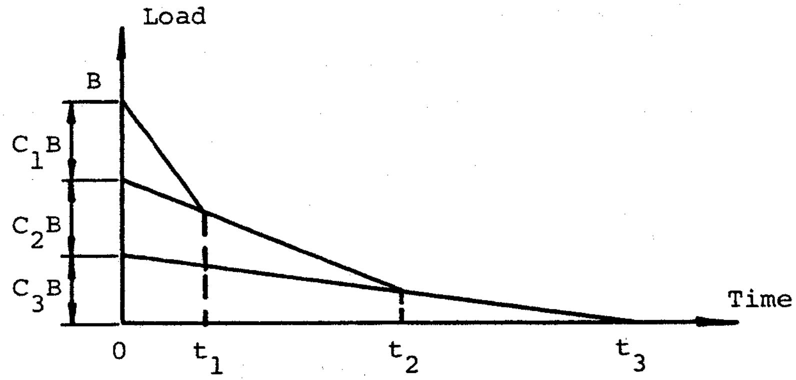


Figure 5-13. Multiple Triangle Approximation of Loading Function

Many airblast loadings of interest can be adequately represented by a two triangle approximation. Equations 5-47 and 5-48 can be combined to obtain a general solution for this case.

$$\frac{\dfrac{C_1 B}{R_m}}{\dfrac{T_N}{\pi t_1}\sqrt{2\mu-1} + \dfrac{1-\dfrac{1}{2\mu}}{1+0.7\dfrac{T_N}{t_1}}} + \frac{\dfrac{C_2 B}{R_m}}{\dfrac{T_N}{\pi t_2}\sqrt{2\mu-1} + \dfrac{1-\dfrac{1}{2\mu}}{1+0.7\dfrac{T_N}{t_2}}} = 1 \tag{5-50}$$